Introduction to Adaptive Optics

Material for a Graduate-Level Course

These slides are put together to support a course in Adaptive Optics. They can be used by any customer of Active Optical Systems, LLC, (AOS) for educational purposes. They are copyrighted by AOS.

What is Adaptive Optics?

• A control system for the spatial phase of a beam of light for
  — Compensating optical aberrations or
  — Changing the spatial phase of a beam of light to create a desired effect.
• Optical equivalent to noise cancelling head-phones
• “De-twinkling the stars”
Typical AO System

Object Being Imaged

Fixed Wavefront

Imaging System

Aberrated Wavefront

Deformable Mirror

Camera

Ideal

Aberrated

Hartmann Wavefront Sensor

Ideal Sensor

Aberrated Sensor

Beam Expander

Secondary Beam Line for HWFS or Target

Membrane Deformable Mirror

Beam Sampler

Lens

Firewire Camera

Diode Laser

Beam Expander

10"

12"
Applications

- Laser Wavefront Control
  - Intensity Profile Shaping
    - Laser Machining
    - Optical Tweezers
    - Medical Applications
  - Atmospheric Aberration Compensation
  - Medical Applications
- Imaging
  - Astronomy
  - Target Inspection
  - Ophthalmology
    - Phoropters

Brief History of Adaptive Optics

- ~200 B.C. - Archimedes & Shield Solar Concentrator
- 1800’s - Newton’s Atmosphere Observations
- 1904 – Hartmann Sensor Invention
- 1953 – H. Babcock
  - Eidophor Compensator (never built)
- 1970’s – Development of Modern Plate-Type Deformable Mirror Technology
- 1971 – Shack-Hartmann Sensor
- 1970’s & 80’s Development and Demonstration of AO Systems
- 1991 – Classified programs published
  - Laser Guide Stars
- 1990’s and 2000’s – Development of MEMs DMs
Common Alternative Technologies

- **“Lucky-Shot” Astronomy**
  - Take thousands of pictures and wait for a better image
- **Deconvolution**
  - Use numerical techniques to remove aberrations
- **Spatial Filtering**
  - Cut out the high spatial frequencies with a filter near a focus
- **Null-Corrector Plates**
  - Compensate static aberrations with a fixed corrector
- **Nonlinear Phase Conjugation**
  - Use high power beam to create a phase compensation in a non-linear medium

Pros and Cons of Conventional Technology

<table>
<thead>
<tr>
<th>Technique</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Luck-Shot” Astronomy</td>
<td>• Relatively Inexpensive with Digital Photography</td>
<td>• Does not scale well (less likely with larger apertures)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Impossible to predict</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Prevents observation of dim objects</td>
</tr>
<tr>
<td>Deconvolution</td>
<td>• Inexpensive</td>
<td>• Requires known complex features</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Requires computationally intensive operations</td>
</tr>
<tr>
<td>Spatial Filtering</td>
<td>• Very cheap</td>
<td>• Wastes significant power</td>
</tr>
<tr>
<td>Null-Corrector Plates</td>
<td>• Simple to use</td>
<td>• Expensive to manufacture</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Requires good alignment</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Point Design (not adjustable)</td>
</tr>
<tr>
<td>Phase Compensation</td>
<td>• Can compensate dynamic and high-frequency aberrations</td>
<td>• Requires very high intensities</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Wastes a large amount of energy</td>
</tr>
</tbody>
</table>
**Barriers to Mass Usage of AO**

<table>
<thead>
<tr>
<th>Barrier</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>Implementation via our unique compact low-cost hardware</td>
</tr>
<tr>
<td>Complexity</td>
<td>Construction of complete active optical systems</td>
</tr>
<tr>
<td>Inertia</td>
<td>AO systems can often relax requirements and increase system functionality</td>
</tr>
</tbody>
</table>

**Technology Progress toward Adoption**

- Polymer membrane & MEMS DMs
- Liquid crystal spatial phase modulators
- Machine vision cameras as wavefront sensors
- Low-cost compact microcontrollers for AO
- Integrated AO systems instead of components
- AO provides a net system gain
  - enabling better performance or new capability for a low system cost
Maxwell’s Equations

- Compiled by James Clerk Maxwell in 1861-2.

1. Electric field comes from a charged particle
   \[
   \nabla \cdot E = \frac{\rho}{\varepsilon_0} : \text{Gauss's Law}
   \]
   \[
   \nabla \cdot B = 0 : \text{Gauss's Law for Magnetism}
   \]
2. No Magnetic Monopoles
3. Changing Magnetic Fields produce an Electric field
   \[
   \nabla \times E = -\frac{\partial B}{\partial t} : \text{Faraday's Law of Induction}
   \]
4. Changing Electric Fields or moving charged particles produce a Magnetic Field
   \[
   \nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} : \text{Ampere's Circuital Law}
   \]

http://en.wikipedia.org/wiki/Maxwell's_equations
Maxwell’s Equations without Charged Particles
(in a dielectric or vacuum medium)

\[ \nabla \cdot E = 0 \]
\[ \nabla \cdot B = 0 \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} \]
\[ \nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \]

Clearly these relationships get significantly simpler...

Deriving The Wave Equation from Maxwell’s Equations

\[ \nabla \times \nabla \times E = -\frac{\partial}{\partial t} \nabla \times B = -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \]

By using the vector identity,
\[ \nabla \times (\nabla \times V) = \nabla (\nabla \cdot V) - \nabla^2 V, \]
we arrive at the wave equation

\[ \nabla (\nabla \cdot E) - \nabla^2 E = -\frac{1}{c_0^2} \frac{\partial^2 E}{\partial t^2} \]

\[ \frac{\partial^2 E}{\partial t^2} = c_0^2 \nabla^2 E \]

http://en.wikipedia.org/wiki/Electromagnetic_wave_equation
Solution of the Wave Equation

- Sinusoidal oscillation at a single frequency.

\[ \frac{\partial^2 E}{\partial t^2} = c_0^2 \cdot \nabla^2 E \]

- We are going to ignore the B field solution here.

\[ E(r, t) = E_0 \cos(\omega t - k \cdot r + \phi_0) \]

Conclusions from Wave Equation

- Light is an electromagnetic wave that is
  - Monochromatic
  - Sinusoidal

- This is directly traceable back to Maxwell’s Equations
Introduction to Matlab: Mathematical Operations & Assignments

• In a matrix assignment
  – [] bound the matrix
  – Space is for separating columns
  – ; is for separating rows
• ; is for separating commands without echoing
• , is for separating commands with echoing
• +, -, *, / are basic math operators for add, subtract, multiply divide, but are defined for matrix math
  – [A B; C D] * [A B; C D] = [AA+BC AB+BD; CA+DC CB+DD]
• .* and ./ are for non-matrix operations
  – [A B; C D] .* [A B; C D] = [AA BB; CC DD]
Introduction to Matlab: Functions

- A function can be created by typing `edit foo.m`
- The code at the right is the basic outline for function with two outputs and two inputs.
- The first comment lines are the help for the user — `%` is the comment operator
- `return` forces the return from the function.

```matlab
function [out1,out2]=foo(in1,in2)
    %FOO is a function for doing addition and multiplication in one function
    out1 = in1 + in2;
    out2 = in1 * in2;
    return;
```

Online Resources

- Google
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J. Goodman’s *Fourier Optics* book is a great reference on this material.

Huygens Principle

In 1678 Christian Huygens “expressed an intuitive conviction that if each point on the wavefront of a light disturbance were considered to be a new source of a secondary spherical disturbance, then the wavefront at any later instant could be found by construction the envelope of the secondary wavelets.”

Huygens-Fresnel Principle = Rayleigh Sommerfield Solution
(Superposition of Waves)

- There is a very elegant mathematical derivation of this equation in
  Goodman’s book.
- It is essentially a mathematical expression of Huygen’s conjecture.

\[
E(P_2) = \frac{1}{j\lambda} \int \int E(P_1) \frac{\exp(jkr_{12})}{r_{12}} \cos(\theta) ds
\]

Spherical Wave Portion

J. Goodman, Introduction to Fourier Optics, Ch 4

The Fresnel Approximation

- Expansion of \( \cos \) term into \( r/z \).
  - Assumes \( r_{12} > > \lambda \)
- Binomial Expansion of \( r_{21} \).
  - Approximation:
    \[
    \sqrt{1+b} = 1 + \frac{1}{2} b - \frac{1}{8} b^2 + ...
    \]

J. Goodman, Introduction to Fourier Optics, Ch 4
Application of $r_{12}$ Approximation: Convolution Form

$$E(x_2, y_2) = \frac{e^{jkz}}{j\lambda z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x_1, y_1) \exp \left[ j \frac{k}{2z} \left( (x_2 - x_1)^2 + (y_2 - y_1)^2 \right) \right] dx_1 dy_1$$

This can be expressed as a convolution:

$$E(x_2, y_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x_1, y_1) h(x_2 - x_1, y_2 - y_1) dx_1 dy_1$$

$$h(x, y) = \frac{e^{jkz}}{j\lambda z} \exp \left[ jk \left( x^2 + y^2 \right) \right]$$

J. Goodman, Introduction to Fourier Optics, Ch 4

Application of $r_{12}$ Approximation: Fourier Transform Form

$$E(x_2, y_2) = \frac{e^{jkz}}{j\lambda z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x_1, y_1) \exp \left[ j \frac{k}{2z} \left( (x_2 - x_1)^2 + (y_2 - y_1)^2 \right) \right] dx_1 dy_1$$

This can ALSO be expressed as a Fourier Transform:

$$E(x_2, y_2) = \frac{e^{jkz}}{j\lambda z} e^{j\frac{k}{2z}(x_2^2 + y_2^2)} \left\{ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x_1, y_1) e^{j\frac{k}{2z}(x_1^2 + y_1^2)} \right\} e^{-j\frac{2\pi}{\lambda z}(x_1 x_2 + y_1 y_2)} dx_1 dy_1$$

J. Goodman, Introduction to Fourier Optics, Ch 4
Accuracy of the Approximations

- The most limiting approximation was the binomial expansion where we dropped the $b^2/8$ term.

- This term is $<< 1$ radian when $z^3$ is large.

$$z^3 >> \frac{\pi}{4\lambda} \left[ (x_2 - x_1)^2 + (y_2 - y_1)^2 \right]_{\text{max}}$$

$$z^3 >> \frac{\pi}{4\lambda} r_{\text{max}}^4$$

**Example:**

\[ \lambda = 10^{-6} \text{ m} = 1 \mu\text{m} \]
\[ r_{\text{max}} = 10^{-2} \text{ m} = 1 \text{ cm} \]
\[ z >> \frac{3\pi}{\sqrt{4 \cdot 10^{-6} (10^{-2})^4}} \approx 0.2 \text{ m} = 20 \text{ cm} \]

---

Simple Fourier Propagator & Notation Simplification

\[ E_2 = P \cdot \int \int E_1 \cdot h \cdot dx_1 \cdot dy_1, \quad h = \exp \left( j \frac{k}{2z} \left[ (x_2 - x_1)^2 + (y_2 - y_1)^2 \right] \right) \]

\[ Q_1 = \exp \left( jk \frac{r_1^2}{2z} \right) \]

- Quadratic Phase Factor (QPF): Equivalent to the effect a lens has on the wavefront of a field.

\[ F(U) = \int \int U \cdot \exp \left( -j \frac{k}{z} (x_2 x_1 + y_2 y_1) \right) \cdot dx_1 \cdot dy_1 \]

- Fourier Transform

\[ P = \frac{\exp(jkz)}{jkz} \]

- Multiplicative Phase Factor: Takes into account the overall phase shift due to propagation

\[ E_2 = P \cdot Q_2 \cdot F(E_1 \cdot Q_1) \]
One-Step Fourier Propagator

- Computationally Efficient
  - Single Fourier Transform

- Output Plane Mesh is not controllable
  - Makes for difficulty in cascading propagations

\[
\delta_2 = \frac{\lambda z}{\delta_1}
\]

Convolution Propagator – Two FTs

- Steps:
  - Fourier transform
  - multiplication by the Fourier transformed kernel
  - an inverse Fourier transform

- Advantage:
  - Allows control of the mesh spacing

- Remaining Question for FFT Implementation:
  - What mesh size and spacing should be used?

\[
U_2 = P \cdot \int \int U_1 \cdot h \cdot dx_1 \cdot dy_1
\]

\[
h = \exp \left( j \frac{k}{2z} (x_2 - x_1)^2 + (y_2 - y_1)^2 \right)
\]

\[
F(h) = H = \exp \left[ -j \pi \lambda z (f_x^2 + f_y^2) \right]
\]

\[
U_2 = P \cdot F^{-1} \left( F(h) \cdot F(U_1) \right)
\]

\[
= P \cdot F^{-1} \left( H \cdot F(U_1) \right)
\]
Fraunhofer Propagation (Far-Field)

- If we assume that the propagation distance is very large, then the quadratic phase factor terms can be eliminated.

\[
E(x_2, y_2) = \frac{e^{\frac{j k}{2 z} (x^2 + y^2)}}{j k z} \int_{-\infty}^{\infty} \int E(x_1, y_1) \left( \exp \left[ -j \frac{2 \pi}{\lambda z} (x_1 x_2 + y_1 y_2) \right] \right) dx_1 dy_1
\]

\[
E_2 = \frac{O_2}{j k z} \cdot F(E_1)
\]

Validity Region

\[
z \gg k (x^2 + y^2)_{\text{max}}^2 = \pi r^2_{1, \text{max}}^2
\]

Validity Example

\[
r_{1, \text{max}} = 10^{-3} \text{ m} = 1 \text{ mm}
\]

\[
\lambda = 10^{-6} \text{ m} = 1 \mu \text{m}
\]

\[
z \gg 3.14 \text{ km}
\]

Numerical Modeling of Propagation

- The basic idea is to create a grid of samples of the field and then operate on the grid.

```matlab
>> Dap=1.0; nxy=128; dxy=Dap*2/nxy; x=(-nxy/2:1:nxy/2-1)*dxy;
>> [xx,yy]=meshgrid(x,x); rr=sqrt(xx.^2+yy.^2); ap = (rr<Dap/2);
>> figure; imagesc(ap); axis image;
>> Eout = fftshift(fft2(ap)); figure; imagesc(abs(Eout)); axis image;
```
Numerical Modeling of Propagation 2: Adding More Guardband

• The basic idea is to create a grid of samples of the field and then operate on the grid.

```matlab
Dap=1.0; nxy=128; dxy=Dap*6/nxy; x=(-nxy/2:1:nxy/2-1)*dxy;
[xx,yy]=meshgrid(x,x); rr=sqrt(xx.^2+yy.^2); ap = (rr<Dap/2);
figure; imagesc(ap); axis image;

Eout = ifftshift(fft2(ap)); figure; imagesc(abs(Eout)); axis image;
```

Convolution Propagation in Matlab

```matlab
Dap=1.0; nxy=128; dxy=Dap*6/nxy; x=(-nxy/2:1:nxy/2-1)*dxy; [xx,yy]=meshgrid(x,x);
rr=sqrt(xx.^2+yy.^2); ap = (rr<Dap/2);

figure; imagesc(ap); axis image;

du = 1/(nxy*dxy); u=[0:nxy/2-1 -nxy/2:-1]*du;[ux,uy]=meshgrid(u,u);ur=sqrt(ux.^2+uy.^2);
wavelength = 1e-6; k=2.0*pi/wavelength; z=300e3; H = exp(-i*2*pi^2*ur.^2*z/k);

f1=ifft2(fft2(ap) .* H);
figure; imagesc(abs(f1)); axis image;
```

\[ \lambda=1\mu m, Z=300\text{km} \]
Adequate Phase Sampling

- In most situations, the most rapidly varying part of the field is the QPF.
- In a complex field, the phase is reset every wavelength or $2\pi$ radians.
- To achieve proper sampling, sampling theory dictates that we need two samples per wave.
Mesh Sampling: Angular Bandwidth

\[ \theta_{\text{max}2} = \frac{D_1}{z} \]

\[ \theta_{\text{max}1} = \frac{D_2}{z} \]

\[ \frac{D_1}{z} \leq \frac{\lambda}{2\delta_2} \]

\[ \delta_2 \leq \frac{z\lambda}{2D_1} \]

\[ \frac{D_2}{z} \leq \frac{\lambda}{2\delta_1} \]

\[ \delta_1 \leq \frac{z\lambda}{2D_2} \]

Virtual Adjacent Apertures – “Wrap Around”

- Now that we know the mesh sampling intervals (\( \delta_1 \) and \( \delta_2 \)), we need to know how big a mesh we need to use to accurately model the diffraction.
- The Fourier transform assumes a repeating function at the input.
  - This means that there are effective virtual apertures on all sides of the input aperture.
- We need a mesh large enough that these virtual adjacent apertures do not illuminate our area of interest.
  - This allows us to avoid “wrap-around” by using a guard band.
**Mesh Size: Avoid “Wrap-Around”**

**Fully General Result**

\[
N \geq \frac{D_1}{2\delta_1} + \frac{D_2}{2\delta_2} + \frac{z\lambda}{2\delta_1\delta_2}
\]

**NOTE:** Drawn for \(\delta_1/\delta_2 = D_1/D_2\) and maximum mesh spacing.

---

**Mesh Determination Rules of Thumb**

**Mesh Sample Spacing**

\[
\delta_2 \leq \frac{z\lambda}{2D_1} \quad \text{and} \quad \delta_1 \leq \frac{z\lambda}{2D_2}
\]

Approximation: Mesh spacing should be bigger than half the diffraction limited radius from the other end.

**Mesh Size**

\[
N \geq 16N_{f_{\text{eff}}} = 16\frac{r_1r_2}{\lambda z}
\]

for maximum \(\delta_1\) and \(\delta_2\)

Approximation: Mesh size should be bigger than 16 times the effective Fresnel number.

---

Determining Fourier Propagation Mesh Parameters for Complex Optical Systems of Simple Optics

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Introduction

• Wave-optic mesh parameters can be uniquely determined by a pair of limiting apertures separated by a finite distance and a wavelength.

\[ D_1 \quad z \quad D_2 \]

\[ \lambda \]

• An optical system comprised of a set of ideal optics can be analyzed to determine the two limiting apertures that most restrict rays propagating through the system using field and aperture stop techniques.
Definitions of Field & Aperture Stop

- **Aperture Stop** = the aperture in a system that limits the cone of energy from a point on the optical axis.

- **Field Stop** = the aperture that limits the angular extent of the light going through the system.
  
  – NOTE: All this analysis takes place with ray optics.
Procedure for Finding Stops 1/3

Find the location and size of each aperture in input space.

1. Find the ABCD matrix from the input of the system to each optic in the system.

2. Solve for the distance \( z_{\text{image}} \) required to drive the B term to zero by inverting the input-space to aperture ray matrix.
   - This matrix is the mapping from the aperture back to input space.

3. The A term is the magnification \( M_{\text{image}} \) of the image of that aperture.

\[
M_i = M_{\text{input-space to aperture}}^{-1} = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}
\]

\[
\begin{bmatrix} 1 & z_{\text{image}} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} = \begin{bmatrix} M & 0 \\ C & 1/M \end{bmatrix}
\]

\[
z_{\text{image}} = -\frac{B}{D}
\]

\[
M_{\text{image}} = C \cdot z_{\text{image}} + A
\]

Procedure for Finding Stops 2/3

2. Find the angle formed by the edges of each of the apertures and a point in the middle of the object/input plane. The aperture which creates the smallest angle is the image of the aperture stop or the entrance pupil.

\[
\begin{align*}
&L1 & &15 \\
&L2 & &-50 \\
&A1 & &5 \\
&A2 & &1 \\
& & &0 \\
& & &150 \\
& & &15
\end{align*}
\]
Procedure for Finding Stops 3/3

2. Find the aperture which most limits the angle from a point in the center of the image of the aperture stop in input space. This aperture is the field stop.

Example: Fourier Propagation

D1 = 1 mm, D2 = 15 mm, λ = 1 μm, z = 0.15 m
Minimal Mesh = 400 x 9.375 μm = 3.75 mm
Example System Modeled

Optical System

A1

D=1

f=100

L1

D=15

L2

D=15

f=100

D=5

A2

Input

150

Plane 2

200

Plane 3

50

Plane 4

N=1024, δ=6.6 μm

Over-Sampled
N = 512, \( \delta = 9.4 \) \( \mu \text{m} \)

Plane 2 Plane 3 Plane 4

Minimal Sampling

N = 256, \( \delta = 13.3 \) \( \mu \text{m} \)

Under Sampled

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Conclusions

• We have devised a procedure to reduce a complex system comprised of simple optics into a pair of the most restricting apertures using the concepts of field stop and aperture stop.
• With these two apertures, a wavelength, and a distance, we can determine the mesh parameters for this system.
• Limitation: Does not include possibility of soft-edged apertures or aberrations, but they can be added.

Gaussian Beam & ABCD Propagation

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**Gaussian Beam Propagation Modeling**

Gaussian beam field and intensity patterns

Size and Wavefront
Radius of Curvature
Rayleigh Range (collimated beam range)
Gouy or “Extra” Phase

\[ E = E_0 \frac{w_0}{w(z)} \exp \left( -\frac{r^2}{w^2(z)} \right) \exp \left( -jkz - jk \frac{r^2}{2R(z)} + jG(z) \right) \]

\[ I = I_0 \frac{w_0}{w(z)} \exp \left( -2 \frac{r^2}{w^2(z)} \right) \]

\[ w(z) = w_0 \sqrt{1 + \left( \frac{z}{z_R} \right)^2} \]

\[ R(z) = z \left[ 1 + \left( \frac{z_R}{z} \right)^2 \right] \]

\[ z_R = \frac{\pi w_0^2}{\lambda} \]

\[ G(z) = \tan^{-1} \left( \frac{z}{z_R} \right) \]

**Introduction - Ray Matrices**

- The most common ray matrix formalism is 2x2
  - a.k.a. ABCD matrix
- It describes how a ray height, \( x \), and angle, \( \theta_x \), changes through a system.

\[
\begin{bmatrix}
  x' \\
  \theta_x'
\end{bmatrix} =
\begin{bmatrix}
  A & B \\
  C & D
\end{bmatrix}
\begin{bmatrix}
  x \\
  \theta_x
\end{bmatrix}
\]

\[ x' = Ax + B\theta_x \]

\[ \theta_x' = Cx + D\theta_x \]

Small angle approximation (\( \sin \theta \approx \theta \))
### 2x2 Ray Matrix Examples

#### Propagation

\[
x' = x + \theta_x L
\]

\[
\begin{bmatrix}
x' \\
\theta_x'
\end{bmatrix} =
\begin{bmatrix}
1 & L \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
\theta_x
\end{bmatrix}
\]

#### Lens

\[
\theta_x' = \theta_x - x / f
\]

\[
\begin{bmatrix}
x' \\
\theta_x'
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
-1 / f & 1
\end{bmatrix}
\begin{bmatrix}
x \\
\theta_x
\end{bmatrix}
\]

### 5x5 Formalism

- We use a 5x5 ray matrix formalism as a combination of the 2x2, 3x3, and 4x4.
  - Previously introduced by Paxton and Latham
- Allows modeling of effects not in wave-optics.
  - Image Rotation
  - Reflection Image Inversion

This formalism is beyond this course, but is nice to know about for future work.
Gaussian Beam Complex Beam Parameter

- Gaussian beam propagation through a system can be modeled with ABCD ray matrix theory by operating on the complex beam parameter.

\[ q(z) = z + q_0 = z + jz_R \]

\[ \frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{\lambda}{\pi w^2(z)} \]

\[ q_2 = \frac{Aq_1 + B}{Cq_1 + D} \]

Seidel and Zernike Polynomials

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Common Sources of Aberrations

• It is fairly easy to polish optics into spherical surfaces, but this is not the ideal lens shape.
• Even with good lenses, it can be difficult to obtain ideal alignment.
• Misalignment and non-ideal shape cause distortions on a beam of light propagating through a system.

[Image source: en.wikipedia.org/wiki/Optical_telescope#The_five_Seidel_aberrations]

Seidel Aberrations

• Spherical Aberration
  – Makes blurry images that cannot be taken out by adjusting focus
  – 4th Order Radial Term
• Coma
  – 3rd Order Radial Term
  – Named for a “comet” shape that it produces
• Astigmatism
  – Saddle or Cylinder+Parabola
  – 2nd Order Radial Term
• Curvature of Field
  – Image is in focus on a curved surface, not on a flat plane
• Distortion
  – Barrel or Pincushion

Zernike Polynomials

- Infinite set of polynomials
- Represent the Seidels in the low-order terms.

\[
R_n^m(r) = \sum_{k=0}^{(n-m)/2} \frac{(-1)^k (n-k)!}{k!(n+m)/2-k!((n-m)/2-k)!} r^{n-2k}
\]

\[
Z_n^m(r, \theta) = R_n^m(r) \cos(m\theta)
\]

\[
Z_n^{-m}(r, \theta) = R_n^m(r) \sin(m\theta)
\]
Matlab Code

% close all; clear all; clc;
ppt=1;

Dap = 1e-2;
nxy = 512;
Nzern = 15;
wavelength =1e-6;
dxy = Dap*8/nxy;
x=-(-nxy/2:1:nxy/2-1)*dxy;
[xx,yy]=meshgrid(x,x);
rr = sqrt(xx.^2 + yy.^2);
ap = (rr<Dap/2);
z = 100e3;

for ii=1:Nzern;
    v=zeros(1,Nzern);
    v(ii)=0.3*wavelength;
    phs=ZernikeCompose(v,nxy,dxy,Dap/2);

E = ap .* exp(1j*(2.0*pi/wavelength)*phs);

figure;
subplot(1,2,1); imagesc(x,x,phs);
axis image; axis(Dap.*[-1 1 -1 1]);
colorbar; title('Phase');
Eff = fftshift(fft2(E));
dxff = wavelength / dxy;
Dff = 2.44 * wavelength / Dap;
xff = (-nxy/2:1:nxy/2-1).*dxff;
subplot(1,2,2); imagesc(xff,xff,abs(Eff));
axis image; axis(0.4.*[-1 1 -1 1]);
title('Far-Field Magnitude');

if (ppt)
    ToPPT(gcf,'Far-Field Magnitudes of Zernikes',[5 3 ii],ii==1);
end;
end;
Interferometer Architectures

- There are many variations on interferometer designs, but there are three common architectures.

http://en.wikipedia.org/wiki/Michelson_interferometer
http://en.wikipedia.org/wiki/Mach-Zehnder_interferometer
Requirements for Interferometers

• Coherence
  – Easiest with a long coherence length (narrow bandwidth)
  – White light is possible, but very difficult

• Reference Arm
  – For AO, is often self-referencing

Self-Referencing Interferometer

• An input beam of light is filtered in one arm to remove all the aberrations and then interfered with the unfiltered beam.
Derivation of Field Interference

\[ E_1 = A_1 \exp(j\phi_1) \]
\[ E_2 = A_2 \exp(j\phi_1) \]
\[ E_t = E_1 + E_2 \]
\[ I_t = E_t \cdot E_t^* = A_1 e^{j\phi_1} \cdot A_1 e^{-j\phi_1} + A_2 e^{j\phi_2} \cdot A_2 e^{-j\phi_2} + A_2 e^{-j\phi_2} A_1 e^{j\phi_1} + A_2 e^{-j\phi_2} A_1 e^{j\phi_1} \]
\[ I_t = I_1 + I_2 + A_1 A_2 e^{j(\phi_1 - \phi_2)} + A_1 A_2 e^{-j(\phi_1 - \phi_2)} \]
\[ = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\phi_1 - \phi_2) \]

Interference of Zernikes

Matlab Code

```matlab
% close all; clear all; clc;
ppt=1;

Dap = 1e-2;
nxy = 512;
Nzern = 15;
wavelength =1e-6;
dxy = Dap*2/nxy;
x=(-nxy/2:1:nxy/2-1)*dxy;
[xx,yy]=meshgrid(x,x);
rr = sqrt(xx.^2 + yy.^2);
ap = (rr<Dap/2);
z = 100e3;

for ii=1:Nzern;
    v=zeros(1,Nzern);
    v(ii)=wavelength;
    phs=ZernikeCompose(v,nxy,dxy,Dap/2);
```
Matlab Code 2

E = ap .* exp(1j*(2.0*pi/wavelength)*phs);
Et = ap + E; I = Et .* conj(Et);
figure;
subplot(1,2,1); imagesc(x,x,phs);
axis image; axis(Dap.*[-1 1 -1 1]);
colorbar; title('Phase');
subplot(1,2,2); imagesc(x,x,I);
axis image; axis(Dap.*[-1 1 -1 1]);
colormap(gray);
title('Interference');
drawnow;

if (ppt)
    ToPPT(gcf,'Interference',[5 3 ii],ii==1);
end;
end;

Phase Shifting for Phase Extraction

• With precise knowledge of the field amplitudes, phase difference extraction could be accomplished with arccosine.

\[
\tan(\Delta \phi) = \frac{I_4 - I_2}{I_1 - I_3}, \text{where}
\]

\[
I_1 = I_A + I_B + 2\sqrt{I_A I_B} \cos(\Delta \phi + 0)
\]
\[
I_1 = I_A + I_B + 2\sqrt{I_A I_B} \cos(\Delta \phi + \pi / 2)
\]
\[
I_1 = I_A + I_B + 2\sqrt{I_A I_B} \cos(\Delta \phi + \pi)
\]
\[
I_1 = I_A + I_B + 2\sqrt{I_A I_B} \cos(\Delta \phi + 3\pi / 2)
\]

• Real-world variations prevent this exactly, but phase shifting interferometry offers a simple solution.

D. Malacara, Optical Shop Testing
Implementations of Phase Shifting

- **Temporal**
  - Mechanical Motion (PZT)
  - Electro-Optic Modulator in a Fiber used in self-referencing

- **Spatial**
  - Fixed phase shift between four different interference patterns
  - Fast, but costs light
Hartmann Wavefront Sensors

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Hartmann Sensors

• A Hartmann sensor is an array of apertures in front of an imager
• Originally holes drilled in a plate (1904) and photographic film
• Moved to lens arrays for optical efficiency in 1971 (Roland Shack and Ben Platt)
• Moved to CCDs around 1990.
• Moved to CMOS detectors around 2000.
Theory of Operation

The Hartmann sensor measured the local derivative of the phase.

Hartmann Wavefront Reconstruction

- **Southwell Reconstruction**: Iterative method in which the phase is calculated at each point in the field from the local average of the measured derivatives

  \[
  \phi(i, j) = \phi(i-1, j) + \frac{\frac{d\phi}{dx}(i-1, j) + \frac{d\phi}{dx}(i, j)}{2} \Delta_1
  \]

  \[
  \phi_2(i, j) = \phi(i+1, j) + \frac{\frac{d\phi}{dx}(i+1, j) + \frac{d\phi}{dx}(i, j)}{2} \Delta_2
  \]

  \[
  \phi_3(i, j) = \phi(i, j-1) + \frac{\frac{d\phi}{dx}(i, j-1) + \frac{d\phi}{dx}(i, j)}{2} \Delta_3
  \]

  \[
  \phi_4(i, j) = \phi(i, j+1) + \frac{\frac{d\phi}{dx}(i, j+1) + \frac{d\phi}{dx}(i, j)}{2} \Delta_4
  \]

- **Matrix Methods**: A matrix can be used to calculate the derivative from the phase. Inverting this matrix and multiplying by the derivatives produces the phase.

  \[
  \nabla \phi = M \cdot \phi
  \]

  \[
  \phi = M^{-1} \cdot \nabla \phi
  \]

  \[
  \phi(i, j) = \text{mean}(\phi_1, \phi_2, \phi_3, \phi_4)
  \]
Challenge: Slope Dynamic Range Limits

- One challenge with using a Hartmann sensor is the potential for large slope to move a spot into an adjacent sub-aperture.

Phase Diversity / Curvature Sensors
Intensity Transport:

Light will concentrate as it propagate with positive wavefront curvature.

\[-k \frac{\partial I}{\partial z}(r) = I(r) \nabla^2 \phi(r) + \nabla I(r) \cdot \nabla \phi(r)\]

Uniform Intensity

\[\nabla^2 \phi \approx -\frac{k}{\Delta z} \frac{\Delta I}{I_{ave}}\]

Phase Diversity vs. Curvature Wavefront Sensing

• Phase Diversity is a technique for extracting phase from two intensity profiles, one or both of which are near the focal plane.

• Curvature wavefront sensing is a specific kind of phase diversity wavefront sensing which typically looks at images on either side of a focal plane which are conjugate to near pupil-plane images.

Curvature Wavefront Sensor Construction

Note: This optical configuration produces asymmetry of image planes to lens location.

Roddier’s Curvature AO Concept (1988)

Curvature sensing and compensation: a new concept in adaptive optics
François Roddier
National Optical Astronomy Observatories, Advanced Development Program, P.O. Box 26732, Tucson, Arizona

The simplest feedback loop would be to apply on a membrane mirror a pressure distribution $P(r)$ proportional to the wavefront sensor signal described by Eq. (5). According to Eq. (2), at the equilibrium position the membrane surface $z'(r)$ is given by

$$\nabla^2 z'(r) \propto P(r).$$

Membrane deformable mirrors are curvature devices that can receive scaled commands directly from a curvature wavefront sensor without a control matrix.
Motivation: Faster/Better AO

• Membrane DMs are particularly well suited to direct curvature sensor control.
  – Should be faster than traditional matrix-based AO
  – Other DMs may be as well...

• Phase Diversity & Curvature WFS have other advantages
  – Wide FOV
  – Wide Spectral Bandwidth

Computational Complexity Comparison

<table>
<thead>
<tr>
<th>AO Type</th>
<th>Approximate Floating-Point Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>CWFS AO</td>
<td>96</td>
</tr>
<tr>
<td>HWFS AO</td>
<td>3814</td>
</tr>
</tbody>
</table>

Even with only 32 actuators, direct curvature control can be MUCH computationally faster.
Spatial Phase Modulators

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Plate-Type Deformable Mirrors

- DM Plate Surface with HR Coating
- Actuators
- Base Plate
Invention of the DM

• One of the earliest DM patents is the Fienleib patent filed in 1973.
• Until recently most of the development was concentrated in a strain from Itek, United Technologies, Xinetics, and Northrop Grumman.
• Since then, the Optical Physics Company, OKO, and Active Optical Systems have developed plate-type DM technology.
Plate DM Influence Function Shape

- The influence function shapes of plate-type DMs are very localized, except at the unbound edges.
- This gives good spatial frequency response.
- The typical maximum response is $1/2a$ where $a$ is the actuator spacing.

Plate DM Typical Performance Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuator Spacing</td>
<td>5-7 mm</td>
</tr>
<tr>
<td>Actuator Count</td>
<td>7 to 1000+</td>
</tr>
<tr>
<td>Inter-Actuator Throw</td>
<td>2 microns</td>
</tr>
<tr>
<td>Total Throw</td>
<td>4 to 18 microns</td>
</tr>
<tr>
<td>Actively Flattened Surface Figure</td>
<td>20 nm</td>
</tr>
<tr>
<td>Typical Cost</td>
<td>$1k/actuator</td>
</tr>
<tr>
<td>Coating Quality</td>
<td>Best</td>
</tr>
</tbody>
</table>
Membrane Deformable Mirrors

- Metal membrane DMs were published by Grosso and Yellin (1977).
- A MEMS version was first published by L. M. Miller & T. Kenny (1993) from JPL.
- Huge (1m diameter) membrane DMs have been manufactured by SRS.
- Today Active Optical Systems and OKO make commercial membrane DMs.


Prior Polymer DM Results

Low-Cost Polymer Deformable Mirrors

Conductor-Coated Optical Membrane

Spacers

Electrostatic Pad Array

Polymer Deformable Mirror

Business Card
Membrane DM influence functions are not localized, but typically much larger amplitude.
Membrane DM Typical Performance Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuator Spacing</td>
<td>0.5 to &gt;10 mm</td>
</tr>
<tr>
<td>Actuator Count</td>
<td>1 to 100+</td>
</tr>
<tr>
<td>Inter-Actuator Throw</td>
<td>0.5 microns</td>
</tr>
<tr>
<td>Total Throw</td>
<td>10-20 microns</td>
</tr>
<tr>
<td>Actively Flattened Surface Figure</td>
<td>~20 nm</td>
</tr>
<tr>
<td>Typical Cost</td>
<td>$30 / actuator</td>
</tr>
<tr>
<td>Coating Quality</td>
<td>Good</td>
</tr>
</tbody>
</table>
Bimorph Deformable Mirrors

• This DM architecture was also described in the Feinleib patent from 1973.
• The idea is to provide a local field to a continuous plate of piezoelectric material to warp the surface.
• The warp is not localized, but instead looks much like the influence functions of the membrane DMs.

Commercially available from Night RU in Russia

Bimorph DM Typical Performance Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuator Spacing</td>
<td></td>
</tr>
<tr>
<td>Actuator Count</td>
<td></td>
</tr>
<tr>
<td>Inter-Actuator Throw</td>
<td></td>
</tr>
<tr>
<td>Total Throw</td>
<td></td>
</tr>
<tr>
<td>Actively Flattened Surface Figure</td>
<td></td>
</tr>
<tr>
<td>Typical Cost</td>
<td>$500 / actuator</td>
</tr>
<tr>
<td>Coating Quality</td>
<td>Good</td>
</tr>
</tbody>
</table>
MEMS Deformable Mirrors

- Surface Micromachined MEMS DMs have been investigated by a variety of groups at Boston University (Boston Micromachines), UC Berkeley (IrisAO), Stanford University, Sandia National Labs, etc.
- Today, most of the development is concentrated at IrisAO and Boston Micromachines (BMM).

Common MEMS DM Architectures

**Segmented MEMS DM (IrisAO)**

- Very large throw capability
- Scalable in size
- Zero inter-actuator cross-talk
- Large edge scattering
- <100% fill factor
- Good coating capable

**Three-Layer Architecture MEMS DM (Stanford and BMM)**

- Lower throw capability
- Nearly zero inter-actuator cross-talk
- Scattering from etch holes
- <100% fill factor
- Poor coating capability
MEMS DM Typical Performance Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuator Spacing</td>
<td>100 to 500 microns</td>
</tr>
<tr>
<td>Actuator Count</td>
<td>19 to 1000+</td>
</tr>
<tr>
<td>Inter-Actuator Throw</td>
<td>2 microns BMM, 10+ microns Iris</td>
</tr>
<tr>
<td>Total Throw</td>
<td>2 microns BMM, 10+ microns Iris</td>
</tr>
<tr>
<td>Actively Flattened Surface Figure</td>
<td></td>
</tr>
<tr>
<td>Typical Cost</td>
<td>$500 / actuator</td>
</tr>
<tr>
<td>Coating Quality</td>
<td>Good (Iris), Poor (BMM)</td>
</tr>
</tbody>
</table>

Liquid Crystal Spatial Light Modulator

- Boulder Nonlinear Systems (BNS) and Meadowlark are making high resolution spatial phase modulators from liquid crystal

LC SLM Typical Performance Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuator Spacing</td>
<td>~15 microns</td>
</tr>
<tr>
<td>Actuator Count</td>
<td>up to 512+</td>
</tr>
<tr>
<td>Inter-Actuator Throw</td>
<td>~1 microns</td>
</tr>
<tr>
<td>Total Throw</td>
<td>~1 micron</td>
</tr>
<tr>
<td>Actively Flattened Surface Figure</td>
<td></td>
</tr>
<tr>
<td>Typical Cost</td>
<td>$40 / actuator</td>
</tr>
<tr>
<td>Coating Quality</td>
<td>Good, but lossy absorbing electrodes</td>
</tr>
</tbody>
</table>

Matching Phase Modulators to Applications

- To choose a phase modulator, it is important to consider all the system requirements including:
  - Required Reflective Coating / Acceptable Loss
  - Required Throw & Inter-Actuator Throw
  - Influence Function Shape
  - Actuator Count
  - Cost
Kolmogorov Spectrum
Turbulence

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Outline

• Origins
• Phase Structure Function
• Frozen Flow Approximation
• Greenwood Frequency
• Generation using Probability Density Function
• RMS Wavefront Error after Compensation
Basic Origin of Kolmogorov Spectrum
Turbulence

• Derived from 1948 work of Kolmogorov on the atmosphere
  – Mostly wind velocity variations with space
• Kolmogorov derived a wind velocity structure function that established the statistics.

\[ D_v = \left\langle \left[ v_r (r_i + r) - v(r_i) \right]^2 \right\rangle = C_v^2 r^{2/3} \]
\[ \langle x \rangle = \text{mean} \]

A good reference for this material is Robert Tyson’s Principles of Adaptive Optics.

Kolmogorov Turbulence Theory
Extension to Optical Aberrations

• Tartarskii (et al.) extended this to distribution of the refractive index.
• Further analysis showed that the “largest telescope diameter that is not significantly affected by turbulence” is Fried’s coherence length \((r_0)\).
• This can then be extended to a power spectral density of phase.

\[ D_n (r) = C_n^2 r^{2/3} \]
\[ D_\phi \approx 6.88 \left( \frac{r}{r_0} \right)^{5/3} \]
\[ \Phi(\xi) = \left( \frac{0.023}{r_0^{5/3}} \right) \xi^{-11/3} \]
\( \xi \) is spatial frequency

Robert Tyson’s Principles of Adaptive Optics.
Fried’s Coherence Length (Diameter)

- This is the largest diameter a telescope can be before being significantly impacted by turbulence.
- It also gives a good estimate for DM actuator spacing in big-beam space.

Kolmogorov Wavefront Variance

- The wavefront variance can be determined from the PSD.
- These approximations are good for estimating wavefront error in turbulence.

\[
\sigma^2 = \int \Phi(\xi) d\xi \\
\sigma^2_{\text{uncompensated}} \approx 1.02 \left( \frac{D_{\text{aperture}}}{r_0} \right)^{5/3} \\
\sigma^2_{\text{tilt-compensated}} \approx 0.134 \left( \frac{D_{\text{aperture}}}{r_0} \right)^{5/3} \\
\sigma = \text{RMS Wavefront Error} = \sqrt{\sigma^2}
\]
Marechal Approximation

- Strehl ratio is a good estimate of optical performance on a 0 to 1 scale.
  - Ratio of real to ideal on-axis intensity
- Marechal is a good approximation out to about λ/8 RMS

\[ \text{Strehl Ratio} = \exp\left(-\sigma_{\text{radians}}^2\right) \approx 1 - \sigma_{\text{radians}}^2 \]

Effect of Kolmogorov Turbulence
Greenwood Frequency

• We can assume that the wind operates on the atmosphere in the beam line like a screen that is dragged across the beam.
  – Frozen flow approximation

• The motion of the screen gives you an estimate of the required temporal frequency, called the Greenwood frequency.

\[ f_G = 0.43 \frac{V_{\text{wind}}}{r_0} \]

Estimating Compensation of Kolmogorov Turbulence

<table>
<thead>
<tr>
<th>IF Shape</th>
<th>( \kappa )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>0.23 to .35</td>
</tr>
<tr>
<td>Plate-Type</td>
<td>0.39</td>
</tr>
<tr>
<td>Piston</td>
<td>1.26</td>
</tr>
<tr>
<td>Membrane</td>
<td>0.30</td>
</tr>
</tbody>
</table>

\[ \sigma_{\text{fit}}^2 \approx \kappa \left( \frac{r_s}{r_0} \right)^{5/3} \]

where \( \kappa \) is a fitting parameter that depends on the DM.
Atmospheric Aberration Compensation with DMs

<table>
<thead>
<tr>
<th>IF Shape</th>
<th>$\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>0.23 to 0.35</td>
</tr>
<tr>
<td>Plate-Type</td>
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<tr>
<td>Piston</td>
<td>1.26</td>
</tr>
<tr>
<td>Membrane</td>
<td>0.30</td>
</tr>
</tbody>
</table>

$$\sigma_{dm}^2 = \kappa \cdot \left( \frac{r_s}{r_0} \right)^{5/3}$$

Actuators per $r_0$

Matlab Example: Generating Kolmogorov Turbulence from a PSD

- This code generates a Kolmogorov spectrum phase screen.
- Generation of this screen in Matlab gives me an RMS WFE of 1.88 radians, which is significantly off from the expected 3.86 radians, but this is likely due to a reduced low-order term due to the Fourier generation technique.

```matlab
close all; clear all; clc;
Dap = 1.0;
Dap_over_r0 = 5;
nxy=1024;
dxy = Dap/nxy;
randn('seed',12345);
x=(-nxy/2+1:nxy/2-1);
[xx,yy]=meshgrid(x);
rr = sqrt((xx.^2+yy.^2));
PSD=0.023*rr.^(-11/3);
PSD(nxy/2+1,nxy/2+1)=0;
randNumbs = randn(nxy,nxy) + 1j*randn(nxy,nxy);
PHI = sqrt(PSD) .* randNumbs;
phi = real(fft2(fftshift(PHI)));
r0 = Dap./Dap_over_r0;
Screen = ((nxy.*dxy./r0).^((5/6)) .* phi;
figure; imagesc(Screen); axis image; colorbar;
```
Wavefront Control: Metric Adaptive Optics

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jmansell@aos-llc.com

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Metric AO Outline

• Common Metrics
  – Sharpness, 2nd Moment, RMS Slopes, Beam Shape

• Common Techniques
  – Stochastic Parallel Gradient Descent (SPGD)
  – Guided Evolutionary Simulated Annealing (GESA)
  – Axial Searching
  – Stochastic Axial Searching (SAS)
Common AO System Architectures

Laser Beam Shaping & Metric AO

Imaging & WFS Feedback

Common Metrics

- **Image Sharpness**: Measure of the sharpness of edges in an image or beam.
  
  \[
  \text{Sharpness} = \sum |\nabla I|^2 \quad \text{or} \quad \sum |\nabla I|^4
  \]

- **Second Moment**: Measure of the size of a beam.

- **RMS Slopes** from a Hartmann Sensor.

- **Beam Shape**: RMS difference between a target shape and the measured beam shape.

  \[
  \sigma_t = \sigma_x + \sigma_y
  \]

  \[
  \sigma_x = \sqrt{\frac{\sum_{all}[I(x, y) - I_{\text{threshold}}](x - \bar{x})^2}{\sum_{all}[I(x, y) - I_{\text{threshold}}]}}
  \]
Guided Evolutionary Simulated Annealing (GESA) Algorithm

1. Generate a set of families with one parent and a set of children within an initial Gaussian radial distribution.
2. The best child becomes the parent.
3. Generate new children with less radius.
4. Go to 2.

- Convergence when
  - no/minimal change or
  - set number of iterations

GESA AO Metric

![Graph showing the Average Second Moment (microns) vs Evaluation Number](image)
Stochastic Parallel Gradient Descent (SPGD) Algorithm

1. Start with a point in the error space.
2. Take a step in a random direction to another point.
3. Find the “optimum” position based on the gradient.
4. Repeat to 2
Comparing GESA and SPGD

- **SPGD** is faster in a smooth error space with no local minima.
- **GESA** is less sensitive to local minima.

**Error Function**

**Optimization Variable**

**SPGD AO Metric**

![Graph showing SPGD AO Metric](image)
SPGD Actuator Commands

Laboratory Optimization Results

SPGD

GESA
Other Algorithms

- **Axial Search**: For each degree-of-freedom, search in that axis independent of the others until an optimum is achieved and then move to the next degree-of-freedom.
  - Very simple to create, but very susceptible to local minimum

- **Stochastic Axial Search**: Generate a random search vector with all the degrees-of-freedom as part of the vector. Search in that direction to find an optimum.
  - Simplified variant of SPGD

Wavefront Control:
Matrix-Based Adaptive Optics
Matrix AO Outline

• Simple Tilt Control
• Generation of Control Matrix
  – SVD Inverse with Mode Removal
  – Matlab Demo
• Slope Discrepancy

Simple Tilt Controller

• It is often easier to understand a 1D tilt controller before moving to a highly multi-dimensional AO controller.
  – FSM = fast steering mirror
  – PSD = position sensing device
Matlab Control Example

- This code models a simple 1D tilt controller with integrator control.
- measTilt is the tilt error.
- When multiplied by the gain and added to the FSMTilt the error is compensated.

```matlab
% simple tilt control
close all; clear all; clc;

Kt = 1;
gain = -1.0;
tv = 0:0.02:5;
FSMTilt = 0.0;
cnt = 1;
for t = tv;
    beamTilt(cnt) = sin(2.0*pi/Kt * t);
    measTilt(cnt) = beamTilt(cnt) - FSMTilt;
    FSMTiltRecorded(cnt)=FSMTilt;
    FSMTilt = FSMTilt - gain * measTilt(cnt);
    cnt = cnt + 1;
end;

figure;
plot(tv,beamTilt,'b-'); hold on;
plot(tv,FSMTiltRecorded,'k-');
plot(tv,measTilt,'g-*');
legend('beamTilt','FSM Tilt','Measured Tilt');
```

Matlab Results

Gain=-1.0, dt=0.020
Gain is too high

Gain=-2.0, dt=0.020
Gain is too high

Gain=-0.5, dt=0.020
Gain is Low

Gain=-0.5, dt=0.050
Sample Rate is Low
Extending to 2D Tilt

- In 2D, there may be coupling between axes if there is any rotation between the FSM and the sensor.
- By poking each actuator (x and y for this example) and measuring the tilt on the PSD, we can measure the actuator response. Concatenating them creates a “poke matrix”
- Inverting that matrix, enables 2D control.

```matlab
>> poke = [1 0.1; 0.1 1];
>> ctrl = pinv(poke)
ctrl =
    1.0101   -0.1010
   -0.1010    1.0101
```

Mathematical Basics of AO Control

Generate slope vector & poke matrix:
\[ \nabla \phi = M_{sys} \cdot \text{cmd} \]

Perform Control:
\[ \Delta \text{cmd}_{n} = \left( M_{sys}^{-1} \right) \cdot \nabla \phi_{measured,n} \]
\[ \text{cmd}_{n+1} = \text{cmd}_{n} + \text{gain} \cdot \Delta \text{cmd}_{n} \]
Generation of an AO Poke Matrix

- Poke actuators on a DM and measure the vector of slopes on a Hartmann sensor
- Creates a matrix of size $N_{\text{actuators}} \times 2*N_{\text{sub-apertures}}$
- This can be done in simulation to avoid noise

Notes on AO Geometries

- There are several named AO geometries. The two most common are Fried and Hudgin.
- These geometries are good for optimizing AO performance, but are not necessary for making an AO system work.
- There is a requirement to have more measurements (slopes) than unknowns (actuator positions).
Generation of a Control Matrix

- Once a poke matrix is generated, it needs to be inverted to create a control matrix.
- This inversion can be done using single value decomposition to enable mode removal.

Comments on Single Value Decomposition (SVD)

- SVD is often used as a method for inverting non-square matrices.
- SVD basically creates the eigenvalues and eigenvectors of the matrix times its transpose.
- This produces a set of orthogonal modes in input space (U), output space (V), and the gains of these modes (S, eigenvalues on the diagonal).
- The matrix is the product of these three matrices:

\[ M = U S V' \]
SVD for Matrix Inversion

- Matrix inversion is the product of the decomposition matrices U, S, and V as shown here.
  - Since S is a diagonal matrix, its inverse is only the reciprocal of the diagonal term.
- It is a good idea to look at the gains (diagonal of the S) to see how easily inverted the matrix is.
  - A spread of >100x between the lowest gain and the highest gain is usually a problem.

\[
M^+ = VS^+U^\prime
\]

Matlab Example

```matlab
>> M = randn(18,16);
>> [u,ss,v]=svd(M);
>> sv=diag(ss); gains=sv;
>> svi = 1.0./sv;
>> si = zeros(size(ss,2),size(ss,1));
>> modesRemoved=0; for ii=1:size(svi,1)-modesRemoved; si(ii,ii) = svi(ii); end;
>> Minv=v*si*u';
>> figure; semilogy(gains);
>> figure; subplot(1,2,1);
   imagesc(M); subplot(1,2,2);
   imagesc(Minv);
```
Matrix Control Conclusions

- We have shown how basic control works with a simple tilt loop.
- We have shown how to extend these concepts to AO control.
- We have shown how to generate a control matrix using a single value decomposition.

Advanced Concepts: Current AO State of the Art
Advanced Concepts Outline

• Ophthalmology
  – Imaging (David in Rochester)
  – Phoropter (LLNL, etc.)
• Astronomy
  – Keck, Subaru, etc.
• Other Imaging
  – Microscopy
  – Navy Optical Interferometer
• Directed Energy
  – ABL, AAOL, ATL, etc.
• Beam Shaping
Far-Field Magnitudes of Zernikes

[Series of images depicting Far-Field Magnitudes of Zernikes]
Far-Field Magnitudes of Zernikes

Interference
Interference
Interference
Interference

Phase Shift Interference
Phase Shift Interference

IFs
IFs
**Application: Laser Machining**

**Gaussian**

**Top-Hat / Super-Gaussian**

**Gaussian**

**Top-Hat / Super-Gaussian**

CO₂ laser on black delrin. Gaussian beam (left); flat-top beam (right).

A laser with a round flat-top uniform intensity beam (right) drills a much cleaner hole in magnetic tape than one with a Gaussian beam (left).

Taken from *Laser Focus World* (2006)

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