ACTIVE OPTICAL SYSTEMS

AN031

Achieving the Maximum Absolute Strehl Ratio Difference using Adaptive Optics in Kolmogorov-Spectrum Turbulence

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Abstract

In this application note, I consider some of the implications of AO on Strehl ratio. In some cases, the aberrations are so weak that the AO system will not make any significant change to the Strehl ratio. In other cases, the aberrations are so strong that the marginal gain from AO will be insignificant. We are trying to find the point at which an idealized AO system might have the most impact on system performance.

Introduction

The Marechal approximation (see Born & Wolf's Principles of Optics, 7th (expanded) edition, p. 528) states that the Strehl ratio can be approximated by:

$$SR \approx 1 - \phi_{RMS\_radians}{}^2$$

where $\phi_{RMS\_radians}$ is the root-mean-square (RMS) wavefront error in radians (after scaling by $\frac{2\pi}{\lambda}$). This can be represented also by:

$$SR \approx \exp\left(-\phi_{RMS\_radians}{}^2\right)$$

which is a nicer form that gracefully decays to zero at high aberrations. It is important to note that this approximation is not really valid for large aberrations and should probably only be used to any decent degree of accuracy to around 1 radian.

If we consider the effect of an adaptive optics system as reducing the RMS wavefront variance (which is the RMS wavefront error squared) by a factor α (similar to an error rejection ratio analysis), we can represent the Strehl ratio as

$$SR \approx \exp\left(-\alpha\left(\phi_{RMS}{}^2\right)\right)$$

We can then find that the difference Strehl ratio of the corrected system and the uncorrected system to be maximized when

$$\phi^2 = \frac{\ln(\alpha)}{\alpha - 1}.$$

This is interesting because it shows us where an AO system would have the maximum impact on Strehl ratio. The following table summarized select results using this analysis.

α	ϕ at Maximum SR Difference (radians)	Max SR Difference between compensated and uncompensated
0.67	1.10	0.15
0.5	1.18	0.25
0.33	1.28	0.38
0.25	1.36	0.47
0.125	1.54	0.65
0.1	1.60	0.70
0.01	2.16	0.95

The general conclusion of this analysis is that AO systems are reaching their maximum efficacy when the RMS wavefront error is approximately between 1 and 2 radians. Based on this result and the region of validity of the Marechal approximation, a flaw in this analysis is that the point of maximum Strehl ratio difference is beyond the true applicability range of the approximation, but it still gives us some indication as to where the AO system would generate the most benefit to an optical system.

Extrapolation to Kolmogorov-Spectrum Atmospheric Aberrations

According to Robert Tyson's Principles of Adaptive Optics (2nd Edition, p. 41) the wavefront variance of Kolmogorov spectrum turbulence is given by,

$$\sigma^2_{uncompensated} = 1.02\left(\frac{D}{r_0}\right)^{5/3}$$

$$\sigma^2_{tilt-compensated} = 0.134\left(\frac{D}{r_0}\right)^{5/3}$$

where D is the aperture diameter and r_0 is Fried's coherence length. Using these relationships, we find that AO systems are at their maximum efficacy when the ratio D/r_0 is between 1 and 2.5 for uncompensated turbulence and between 4 and 8 when tilt compensated.

Conclusions

Given the approximations of the analysis we did here, we learned that the largest absolute change to Strehl ratio using an AO system should occur between 1 and 2 radians RMS wavefront error and when the D/r_0 ratio is between 1 and 8. This analysis should be followed-up with real wave-optics analysis for verification, but it gives us a good place to start.