AN024

Using Multiple DMs for Increased Spatial Frequency Response

Author: Justin Mansell
Revision: 12/2/10

Abstract

Some researchers have come to us suggesting that using multiple deformable mirrors in series with interleaved (staggered) actuator patterns would enable them to achieve better spatial frequency response. This application note explores the concept of using two deformable mirrors to increase the spatial frequency compensation capability of the combined system. Generally we find that this technique does not work, but it does increase the compensation capability amplitude. There is one exception, and that is using a DM at an angle to create a projection of the beam onto the DM that will have an increased spatial frequency in one axis.

Model Setup

To test this concept, we assumed perfect imaging between the two DMs such that we could simply combine their influence functions in a single plane. We then assumed a half actuator spacing offset in both axes to create the actuator overlay shown in Figure 1. The actuators were assumed to be a 7x7 grid on a 6 mm pitch. The appendix contains our modeling code for this study.

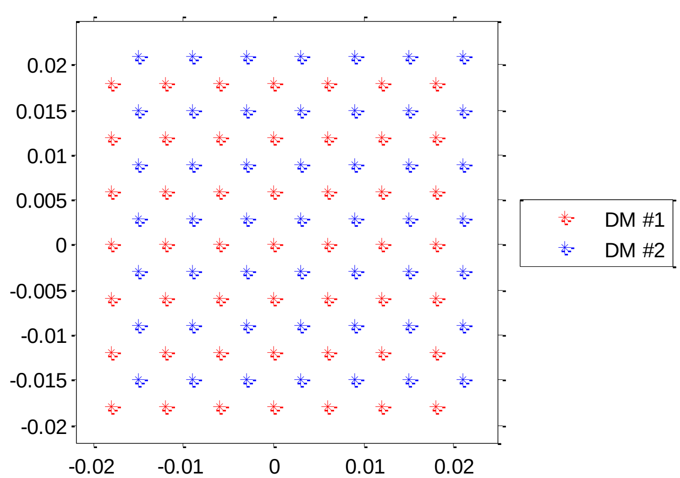

Figure 1 - Overlay of the two sets of DM actuator locations. (Lateral dimensions are meters)

We started our model by generating two sets of Gaussian-shaped influence functions (=exp(-(r/w)^2) with a radius, w, equal to the actuator spacing (6 mm). To test the efficacy of the DM at creating a piston term without excess higher-order waffle, we summed all the influence functions from the non-offset DM. Figure 2 shows the sum of the influence functions from the non-offset DM (DM #1) with the locations of the actuators. The central region of this is very flat as would be expected in a real plate-type DM. This modeling was done on a 128 x 128 pixel grid with 0.36 mm spacing.

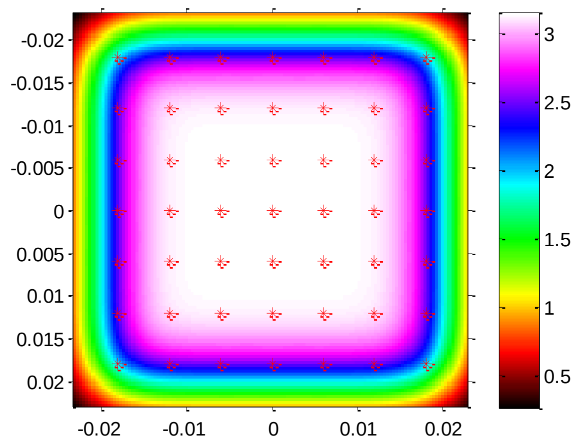

Figure 2 - Sum of all the IFs from the non-offset DM (Lateral dimensions are in meters)

Waffle Pattern Analysis

We first approached this by looking at a waffle pattern since it is the highest spatial frequency pattern a DM can create in both axes. We generated a waffle in both DMs separately and then summed them to attempt to create a higher frequency waffle. To determine spatial frequency response, we Fourier transformed the waffle patterns and examined the Fourier magnitude for each pattern. Figure 3 shows the results of this waffle and Fourier transform analysis.

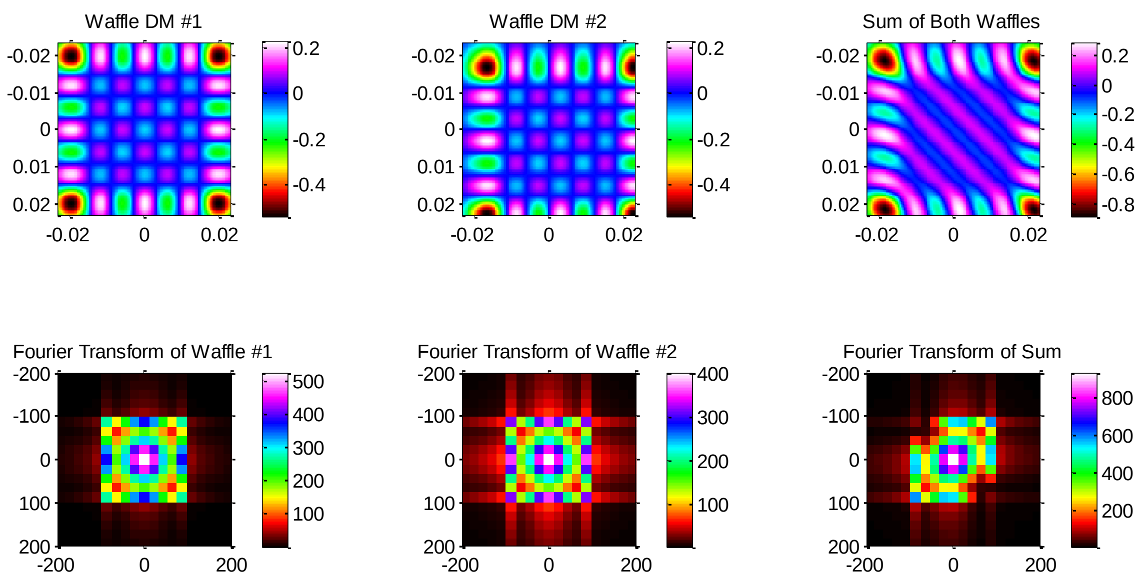

Figure 3 - Results of analysis of the waffle patterns from the two DMs separately and then the sum of the two DM waffle patterns. The Fourier domain graphs are in inverse meter units. The spatial domain is in meter units.

Random Command Spectral Content

To further analyze the spatial frequency response of the DM, we examined a random set of wavefront correction patterns as if generated by reflection from the two staggered DMs. We generated a vector of Gauss-normalized random weights and used them to create a weighted sum of influence functions from both DMs. We then Fourier transformed the surface to determine its spatial frequency content. The amplitude of each of the Fourier transforms from the random realizations was averaged to create a representation of the average spatial frequency content. This average frequency content representation is the square-root of the power spectrum. Figure 4 shows results of this simulation.

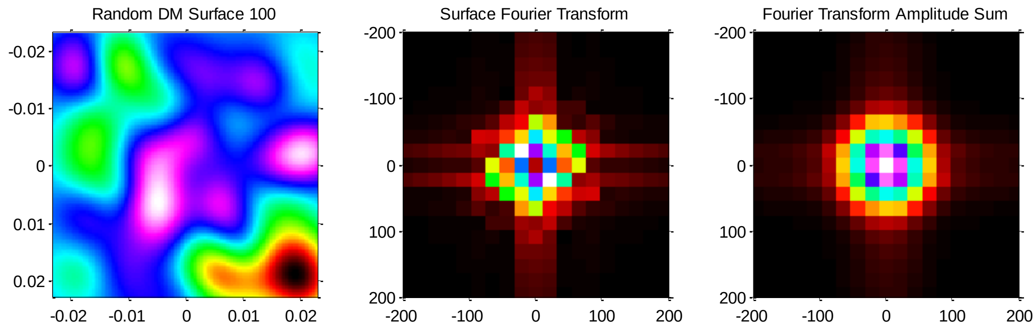

Figure 4 - The 100th random realization of the combined DM surfaces, its individual Fourier transform amplitude, and the average of all the Fourier transform amplitudes from each of the 100 random realizations. The Fourier domain graphs are in inverse meter units. The spatial domain is in meter units.

We repeated this for a single DM as well. Figure 5 shows the results of this simulation.

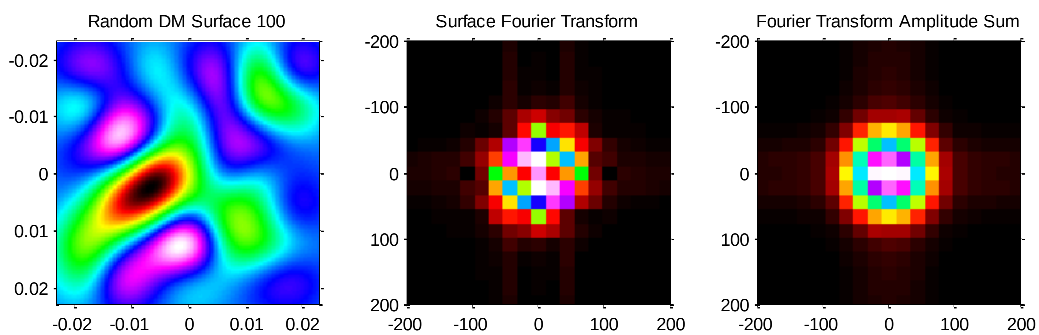

Figure 5 - The 100th random realization of a surface of a single DM, its individual Fourier transform amplitude, and the average of all the Fourier transform amplitudes from each of the 100 random realizations. The Fourier domain graphs are in inverse meter units. The spatial domain is in meter units.

We compared the sqrt(PSD) (aka, average Fourier transform magnitude) of the two results, scaling the two DM result by 1/√2. Figure 6 shows the comparison of the spectral content and a difference between them on the same lateral scale and same colorbar. There is no significant difference in the spectral frequency of the two results.

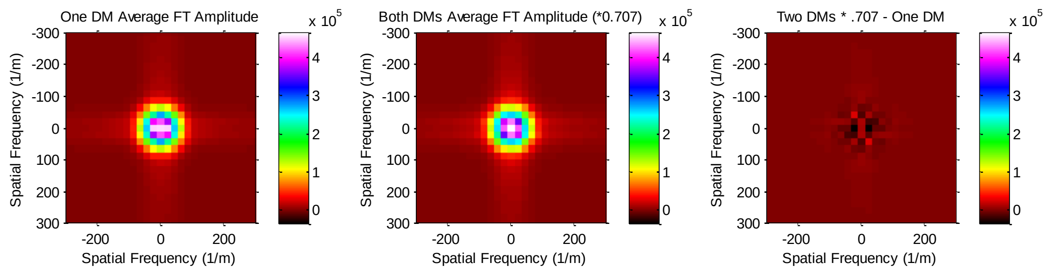

Figure 6 - Comparison of the spectral content of random phase generated by one DM and that generated by two staggered DMs

We repeated this test with 3-mm actuator spacing, the same number of actuators, and the same mesh size. Figure 7 shows this result. We are clearly seeing an increase in the spatial frequency response by moving to a smaller actuator pitch.

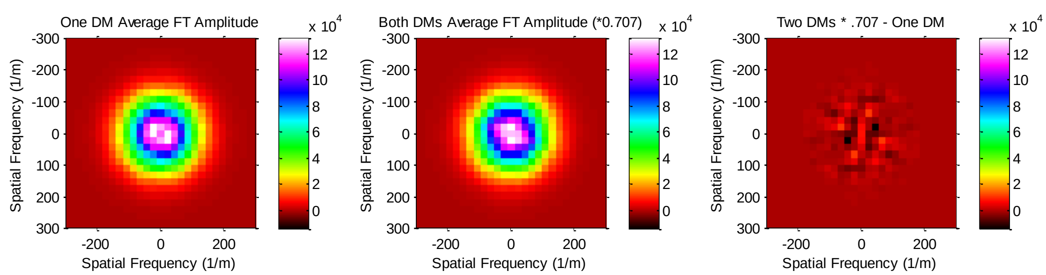

Figure 7 - Comparison of the spectral frequency content of one and two DMs (scaled appropriately) and their difference with a 3-mm actuator spacing

Gaussian Influence Function Argument

Another argument for why staggered DMs in series do not enable higher frequency response can be derived from analysis of the actuator influence function shape itself. Most plate-type deformable mirrors influence functions are fairly accurately represented by a Gaussian. The spatial frequency response of a Gaussian influence functions is again a Gaussian. Staggering DMs creates a sum of Gaussians at an increased density, but does not change the shape of the individual DM's influence function, and thereby does not change the width of the Gaussian in spatial frequency.

Sum of Sinusoids Argument

An alternative argument for why staggered DMs would not produced significantly more spatial frequency response comes from analysis of a sum of sinusoids. DM influence functions are sometimes modeled as a raised-cosine, given by

$$IF(x) = \frac{1+\cos(x)}{2} \qquad -\pi < x < \pi ,$$

where the actuator pitch is π.

The raised-cosine influence function is convenient because the sum of raised cosines is exactly unity. If we then assume that we are trying to create the largest spatial frequency possible with this kind of influence function, we would find that the waffle pattern could be represented by a simple cosine function, or

$$Waffle(x) = \cos(x)$$

When combined with the waffle pattern of a second DM staggered by an arbitrary amount Δ, we can write the combined response as

$$Waffle_{2DMs}(x) = \cos(x) + \cos(x+\Delta).$$

It is clear that for no value of Δ can we achieve a larger spatial frequency than exists on a single DM.

Two DMs at Large Angle of Incidence

Most of the time, DMs are used at a very small angle of incidence (near normal). One way of increasing the spatial frequency of a DM in one axis is to use the DM at a large angle of incidence. In this configuration, the beam projects over more actuators, but the phase modulation depth will be reduced as well. If two DMs can be used at a large angle of incidence, gains can be made in both axes, but again at a cost of total effective throw.

Wavefront Sensor Geometry with Staggered DMs

Most high-speed and light-conscious adaptive optics systems use the Fried geometry, which instructs to place the image of the DM actuators at the corners of rectangular-grid Hartmann wavefront sensor sub-apertures. When using two DMs in series with the actuators staggered by half the actuator spacing in two axes, the Fried geometry considering one DM would place the other DM's actuators at the center of the Hartmann sensor sub-apertures, which is nearly the worst place for them when considering wavefront slope sensitivity.

There are a variety of potential solutions to this problem that would enable adequate sensing. First, in the presence of plenty of signal (not photon starved), the Hartmann sensor could simply be oversampled. If there were 4 sub-apertures (2x2) in the same space as would typically be one in Fried geometry, then the wavefront slope variation associated with each DM actuator could be well-resolved. Figure 8 shows the over-resolved solution.

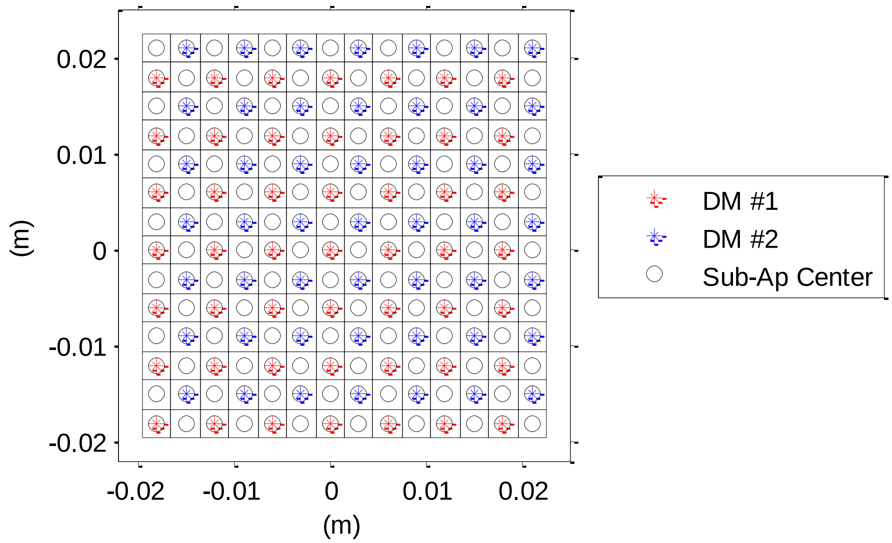

Figure 8 - Over-resolved Hartmann sensor solution for two staggered DMs

Another possible option is to rotate the Hartmann sensor by 45-degrees and then decrease their spacing by a factor of 1/√2. This would place the image of all the actuators at the corners of the sub-apertures again and get the system back into Fried geometry. Figure 9 shows an example of what this geometry might look like.

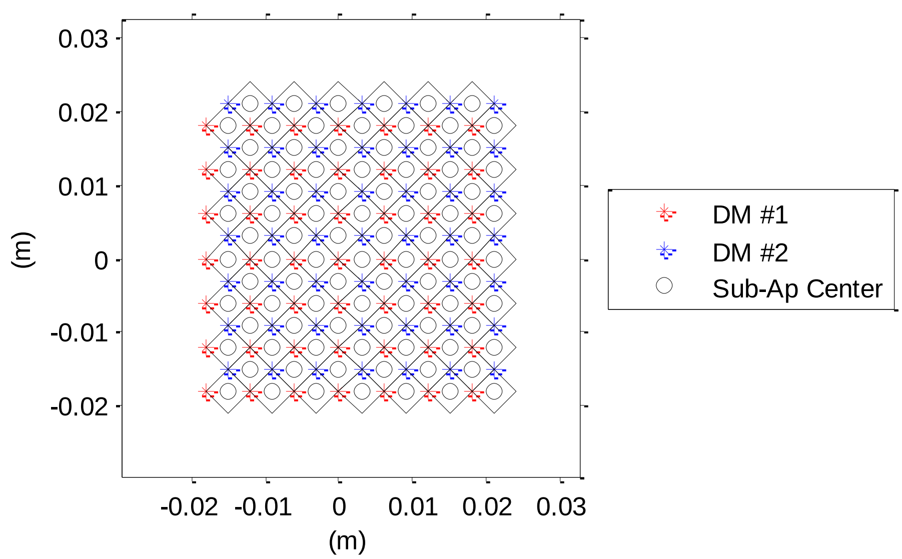

Figure 9 - Fried-like Geometry for Staggered DMs

Appendix: Modeling Code in Matlab

```
% model to test the spatial frequency response of two staggered DMs
close all; clear all; clc;

%setup variables
SF = 1;
dact = 6e-3 / SF;
nxy = 128;
Dap = 7*dact * SF;
realizationsMax=100;

%make actuator grid
xa = (-3:1:3)*dact;
[xxa,yya]=meshgrid(xa,xa);
xact = xxa(:); yact = yya(:);
xact2 = xact + dact/2;
yact2 = yact + dact/2;
Nact = length(xact);

%plot the overlay
figure;
plot(xact,yact,'r*');
hold on;
plot(xact2,yact2,'b*');
axis image;
legend('DM #1','DM #2','Location','EastOutside');

%generate IFs
dxy = 1.1*Dap/nxy;
x = (-nxy/2:1:nxy/2-1)*dxy;
[xx,yy]=meshgrid(x,x);
IFsum = 0.0 .* xx;
for ii=1:Nact;
    rr = sqrt( (xx-xact(ii)).^2 + (yy-yact(ii)).^2 );
    IF{ii} = exp(-1*(rr./(dact)).^2);
    IFsum = IFsum + IF{ii};

    rr = sqrt( (xx-xact2(ii)).^2 + (yy-yact2(ii)).^2 );
    IF2{ii} = exp(-1*(rr./dact).^2);
end;

%plot sum and overlay
figure; imagesc(x,x,IFsum); axis image; colorbar;
hold on;
plot(xact,yact,'r*');

%make a waffle pattern
w = 0.0 .* IF{1};
w2 = 0.0 .* IF2{1};
for ii=1:Nact;
    w = w + (-1).^ii .* IF{ii};
    w2 = w2 + (-1).^ii .* IF2{ii};
end;
figure;
```

```
subplot(2,3,1);
imagesc(x,x,w); axis image; colorbar; title('Waffle DM #1');
subplot(2,3,2);
imagesc(x,x,w2); axis image; colorbar; title('Waffle DM #2');
subplot(2,3,3);
imagesc(x,x,w+w2); axis image; colorbar; title('Sum of Both Waffles');

df = 1/(nxy*dxy);
fx = (-nxy/2:1:nxy/2-1) * df;
subplot(2,3,4); imagesc(fx,fx,abs(fftshift(fft2(w))));
axis image; colorbar; title('Fourier Transform of Waffle #1');
axis([-200 200 -200 200]);
subplot(2,3,5); imagesc(fx,fx,abs(fftshift(fft2(w2))));
axis image; colorbar; title('Fourier Transform of Waffle #2');
axis([-200 200 -200 200]);
subplot(2,3,6); imagesc(fx,fx,abs(fftshift(fft2(w+w2))));
axis image; colorbar; title('Fourier Transform of Sum');
axis([-200 200 -200 200]);

%make one set of all actuator influence functions
c=0;
for ii=1:Nact;
    c=c+1; IFt{c}=IF{ii};
    c=c+1; IFt{c}=IF2{ii};
end;

%% generate a random pattern of influence function sums for 1 DM
zt=0.0 .* IF{1};
randn('seed',-12345);
figure;
for realization = 1:realizationsMax;
    v = randn(1,Nact);
    z=0.0 .* IF{1};
    for ii=1:Nact;
        z = z + v(ii).*IF{ii};
    end;

    %look at fourier transform amplitude
    zffta = abs(fftshift(fft2(z)));
    zt = zt + zffta;

    clf;
    subplot(1,3,1); imagesc(x,x,z); title(sprintf('Random DM Surface
%i',realization));
     axis image;
    subplot(1,3,2); imagesc(fx,fx,zffta); title('Surface Fourier Transform');
     axis image; axis([-200 200 -200 200]);
    subplot(1,3,3); imagesc(fx,fx,zt/realization); title('Fourier Transform
Amplitude Sum');
     axis image; axis([-200 200 -200 200]);
    drawnow;
end;


%% generate a random pattern of influence function sums for BOTH DMs
```

```
zt2=0.0 .* IFt{1};
randn('seed',-12345);
figure;
for realization = 1:realizationsMax;
    NactT = length(IFt);
    v = randn(1,NactT);
    z=0.0 .* IFt{1};
    for ii=1:NactT;
        z = z + v(ii).*IFt{ii};
    end;

    %look at fourier transform amplitude
    zffta = abs(fftshift(fft2(z)));
    zt2 = zt2 + zffta;

    clf;
    subplot(1,3,1); imagesc(x,x,z); title(sprintf('Random DM Surface
%i',realization));
     axis image;
    subplot(1,3,2); imagesc(fx,fx,zffta); title('Surface Fourier Transform');
     axis image; axis([-200 200 -200 200]);
    subplot(1,3,3); imagesc(fx,fx,zt2/realization); title('Fourier Transform
Amplitude Sum');
     axis image; axis([-200 200 -200 200]);
    drawnow;
end;

%% compare results of the random Fourier analysis
figure;
set(gcf,'position',[403         246        1125         420]);
fmax=300;
clear c;
subplot(1,3,1); imagesc(fx,fx,zt);
 axis image; colorbar; title('One DM Average FT Amplitude');
 axis([-fmax fmax -fmax fmax]);
 c(1,:)=caxis;
subplot(1,3,2); imagesc(fx,fx,zt2/sqrt(2));
 axis image; colorbar; title('Both DMs Average FT Amplitude (*0.707)');
 axis([-fmax fmax -fmax fmax]);
 c(2,:)=caxis;
subplot(1,3,3); imagesc(fx,fx,zt2/sqrt(2)-zt);
 axis image; colorbar; title('Two DMs * .707 - One DM');
 axis([-fmax fmax -fmax fmax]);
 c(3,:)=caxis;
max(c)
subplot(1,3,1); caxis([min(c(:,1)) max(c(:,2))]);
xlabel('Spatial Frequency (1/m)'); ylabel('Spatial Frequency (1/m)');
subplot(1,3,2); caxis([min(c(:,1)) max(c(:,2))]);
xlabel('Spatial Frequency (1/m)'); ylabel('Spatial Frequency (1/m)');
subplot(1,3,3); caxis([min(c(:,1)) max(c(:,2))]);
xlabel('Spatial Frequency (1/m)'); ylabel('Spatial Frequency (1/m)');
```