AN021

Analysis of Kolmogorov Spectrum Turbulence

Author: **Justin Mansell**
Revision: **10/19/10**

This application note discusses some observations of simple analysis of Kolmogorov spectrum turbulence. In particular, the question of why the number of DM actuators required to adequately compensate Kolmogorov turbulence is proportional to D/r_0.

Kolmogorov Turbulence Spectrum

We can generate Kolmogorov spectrum turbulence screens based on the power spectral density (PSD), which is well approximated by,

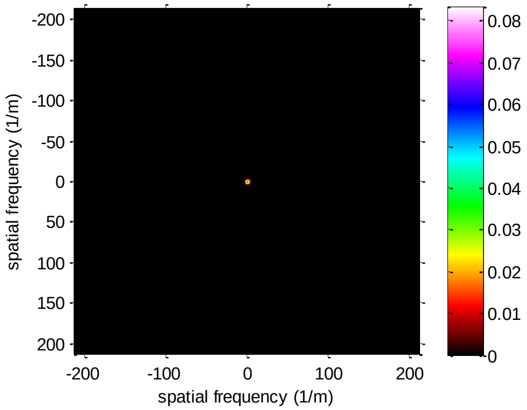
$$\Phi\left(f_s\right) \approx \frac{0.023}{r_0^{5/3}} f_s^{-11/3}$$

where f_s is the spatial frequency and r_0 is the Fried coherence length, which is approximately the largest size of a telescope that is not significantly affected by atmospheric turbulence.[1] The PSD and an example phase screen can be calculated in Matlab with the following code:

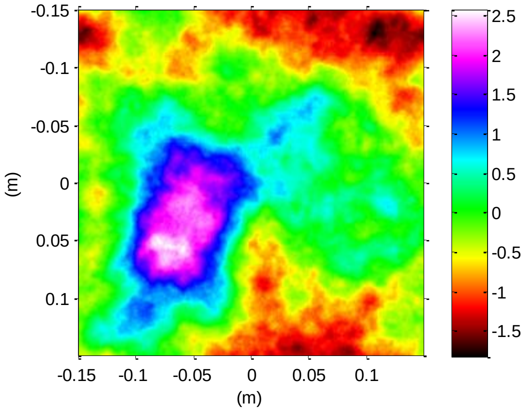
```
%% spectral generation
Dap = 30e-2;
nxy = 256;
r0 = Dap/2;
dxy = Dap/nxy;
df = 1/(2*nxy*dxy);
x = (-nxy/2:1:nxy/2-1) * dxy;
xf = (-nxy/2:1:nxy/2-1) .* df;
[xx,yy]=meshgrid(xf,xf);
rr = sqrt(xx.^2+yy.^2);

%establish spectral density
SD = (0.023/r0^(5/3)) .* (rr).^(-11/3);
SD(nxy/2+1,nxy/2+1)=0; %remove piston
nf; show(xf,xf,SD);
xlabel('spatial frequency (1/m)'); ylabel('spatial frequency (1/m)');

%generate random screen
rndScr = randn(nxy,nxy) + 1j .* randn(nxy,nxy);
phs = real(ifft2(fftshift(sqrt(SD) .* rndScr)*nxy*nxy));
```

```
phs = phs - mean(phs(:));
nf; show(x,x,(phs));
xlabel('(m)'); ylabel('(m)');

RMSphs = sqrt(mean(phs(:).^2))
RMS_notilt_ideal=sqrt(.134) * ((Dap/r0).^(5/6))
RMS_uncomp_ideal=sqrt(1.02) * ((Dap/r0).^(5/6))
```

The following figure shows the PSD calculated using this code.



This figure shows the generated phase.



This FFT technique is known to be poor at generating the low-order terms. There are several techniques that are used to address this. The simplest technique is to generate a phase screen over a much larger area and then clip out a smaller section. The code above is not doing any low-order compensation, so the rms wavefront error is between the tilt-compensated result and the un-compensated result predicted by theory. The results reported by the code are:

RMSphs = 0.8308
RMS_notilt_ideal = 0.6522
RMS_uncomp_ideal = 1.7995

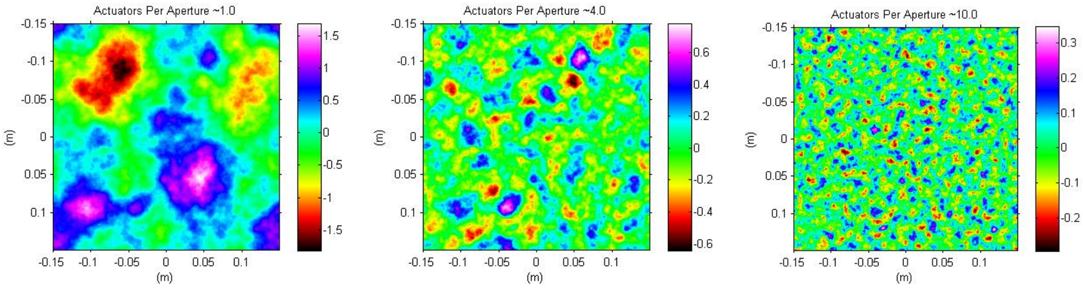
It is important to note that the spatial frequency spectrum does not vary based on Fried's coherence length.

DM as a Spatial Frequency Spectral Filter

To try to model the effect of a DM, we created a spatial filter and applied it to the spectral density before creating the phase screen. This does not model any aspect of throw limitations in the DM, but is a reasonably accurate and quick methodology of studying the effect of adaptive optics on compensating Kolmogorov spectrum turbulence. The filter was implemented as a 10th-order super-Gaussian (where 2nd-order super-Gaussians are simple Gaussians). The code for this was:

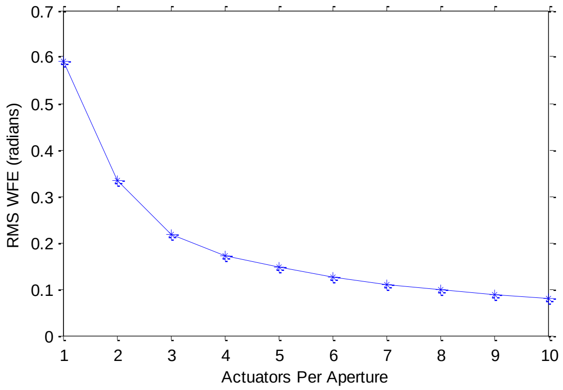
```
%% DM as a spectral filter
for ActuatorsPerAp = 1:1:10
    t = sprintf('Actuators Per Aperture ~%.1f',ActuatorsPerAp);
    dact = Dap/ActuatorsPerAp;
    w = 1/(2*dact);
    filter = 1 - exp(-1.0*(rr./w).^10);
    nf; show(filter);
    if (ppt)
        ToPPT(gcf,t,[1 2 1],1);
    end;

    %generate random screen
    phs = real(ifft2(fftshift(sqrt(SD) .* filter .* rndScr)*nxy*nxy));
    phs = phs - mean(phs(:));
    nf; show(x,x,(phs));
    title(t);
    xlabel('(m)'); ylabel('(m)');
    if (ppt)
        ToPPT(gcf,t,[1 2 2],0);
    end;
end;
```

The following figures show some of the aberrations filtered by the DM filter function.

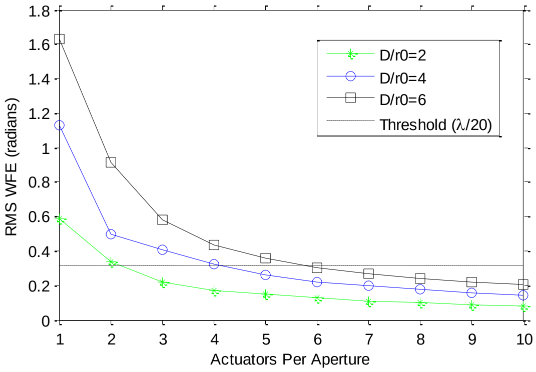


It is clear from these results that the DM filter was removing the low-order terms and the compensated residual is dominated by higher order terms. In fact the compensated residual clearly has increasing spatial frequency content as the number of actuators per aperture increases.

The next figure shows the rms wavefront error as a function of the number of actuators per aperture, which corresponds to the DM filter size in spatial frequency.



From this chart, we can see from this simulation that the residual wavefront error decreases as the number of actuators per aperture diameter. The following figure shows the analysis of RMS wavefront error (residual after filtering with DM).



In this figure we see that the λ/20 cross-over, which corresponds to approximately 90% Strehl ratio, happens when the number of actuators across the aperture equals the D/r_0 ratio. This relationship is what is typically used to initially design the number of actuators required for an AO system.

Conclusion

In this note, we showed that the spatial frequency term in the power spectrum of Kolmogorov spectrum turbulence was independent of the strength, which is represented by r_0. Then we showed that using a spatial frequency filter model of a deformable mirror, we could reduce the turbulence strength to around λ/20 when the number of actuators per aperture diameter was equal to the D/r_0 ratio.

References

---

[1] R. Tyson, Priniciples of Adaptive Optics, 2nd Edition.